Generation of maximally entangled states of two cavity modes

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In this letter we present a scheme for generating maximally entangled states of two cavity modes which enables us to generate complete set of Bell basis states having rather simple initial state preparation. Furthermore, we study the interaction of a two-level atom with two modes of electromagnetic field in a high Q cavity. The two-level atom acts as a control qubit and the two mode electromagnetic field serves as a target qubit. This simple system of quantum electrodynamics provides us experimentally feasible universal quantum logic gates.

Keywords: Three-level system; entanglement; two mode cavity, universal logic gates

Quantum entanglement is a striking nonclassical property of quantum composite systems. In addition to the conceptual problems of reality and locality in quantum physics, quantum entanglement talks about technological aspects of quantum communications, quantum computation and quantum cryptography [1]. There exist situations in which the transition between the upper and lower levels of an atom is mediated by two photons when the energy separation between the levels is close to double the photon frequency. This process and its multiphoton counterparts are important because they can be used to study statistical properties of the optical field [2]. It is therefore desirable to investigate the nonlinear interaction of the three-level atom with the cavity field in the case of the two-photon resonance transitions [21].

Our goal in this letter is to discuss the possibility of obtaining a scheme for generating maximally entangled states in this model and to develop quantum universal logic gates which may generate bimodal entanglements. We have shown that it is possible to generate Bell-type states having rather simple initial state preparation. To reach our goal we have

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to find the exact time dependent expressions for the final state of the system. This can be achieved by solving the Schrödinger equations of motion. In order to realize our suggested scheme in laboratory experiment within microwave region, we may consider slow *Rb* atoms in higher Rydberg states which have life time of the order of few milliseconds [3]. These slow atoms, initially pumped to high Rydberg state, pass through a high-Q superconducting cavity of dimension of a few centimeters with a velocity of around 400 m/s [3,4]. The interaction time of the atom with different cavities can be controlled by using a velocity selector and applying Stark field adjustment in different cavities in order to make the atom resonant with the field for a right period of time [3].

In quantum computation the basic key component is a universal logic gate, which leads to any operation on a qubit. The quantum universal logic gate comprises quantum CNOT gate together with single-qubit Hadamard gate [9, 10, 11, 12]. In addition a quantum phase gate serves as a universal quantum logic gate as well [13]. The quantum controlled-NOT gate has been realized experimentally in Ramsey atomic interferometry, by means of the selective driving of optical resonances of two qubits undergoing a dipole-dipole interaction [14], and by the Bragg scattering of atoms [15]. In this paper we suggest that the interaction of a circular Rydberg atom with a high Q superconducting cavity leads to develop quantum universal logic gate in the electromagnetic field modes. The atom is in resonance with the two modes in the presence and in the absence of Stark field. Thus the Rydberg atom acts as a control qubit whereas the two mode elecrtomagnetic field provides target qubit.

Generation of maximally entangled states between two parties implies existence of the two parties in two different states with equal probability [5]. The generation of maximally entangled states between two cavity modes implies conditional existence of photons in either of the two modes [21]. The maximal entangled state may appear as one photon in one cavity mode and the other mode in vacuum, or as the two cavity modes are either in vacuum state or have one photon each with equal probability. Such maximally entangled states constitute EPR Bell states which provide a complete set of bases. In order to develop the EPR Bell bases we prepare a three level atom in V-configuration, such that, the two upper levels are in superposition state. Hence, in order to generate two electromagnetic field modes entanglement the interaction time of the atom with the cavity is odd integer multiple of half of the Rabi cycles, λ_1 and λ_2. This ensures that the cavity obtains one photon in either of the two modes when atom is detected in ground state after its propagation through the cavity field modes. This implies that we find an entangled state between the two cavity modes. Preparation of the atom in its upper state and letting it interact for an interaction time equivalent to one fourth of Rabi cycle leads

to generate the other two Bell bases. As the atom interacts with the first mode for a quarter of Rabi cycle it finds a laser pulse in resonant with the other transition. Later an interaction for a time half of the Rabi cycle provides another entangled state of the cavity modes where either the two modes are in vacuum or each having one photon. The two kind of entangled states together make complete Bell bases.

We propagate a three-level atom through a cavity which contains initially field modes in vacuum. We may express the three levels as $|a\rangle$, $|b\rangle$, and $|c\rangle$ with their eigen energies as E_a, E_b, and E_c. The dipole transition between the upper two levels, $|a\rangle$ and $|b\rangle$, of the atom is forbidden, whereas transitions from the two upper levels to lower level, $|c\rangle$ are allowed. We consider that frequencies, ω_A and ω_B, of the two modes, A and B, respectively, of cavity field are in resonance with the transition frequencies, such that $\omega_A = (E_a - E_c)/\hbar$ and $\omega_B = (E_b - E_c)/\hbar$. With the help of a Ramsey field we prepare the upper two levels of the atom in linear superposition before it enters the cavity field. We may express the initial state of the system as,

$$\left|\psi^{(0)}\right\rangle = \left[\cos\theta|a\rangle + \sin\theta e^{i\phi}\||b\rangle\right]|0_A, 0_B >, \tag{1}$$

where, ϕ is the relative phase between two atomic states.

We write the interaction picture Hamiltonian in the dipole and rotating wave approximation as

$$H = \hbar g_1\left(a|a\rangle\langle c| + a^{\dagger}|c\rangle\langle a|\right) + \hbar g_2\left(b|b\rangle\langle c| + b^{\dagger}|c\rangle\langle b|\right), \tag{2}$$

where g_1 and g_2 are vacuum Rabi frequencies of the two modes while $a(a^{\dagger})$ and $b(b^{\dagger})$ are the annihilation(creation) operators of the two cavity modes A and B, respectively. The atom-field state vector can be written as

$$\begin{aligned}\left|\psi^{(t)}(A,B)\right\rangle &= \cos\theta\cos g_1t\,|a,0,0\rangle - i\cos\theta\sin g_1t\,|c,1,0\rangle \\ &+ e^{i\phi}\sin\theta\cos g_2t\,|b,0,0\rangle - ie^{i\phi}\sin\theta\sin g_2t\,|c,0,1\rangle\,. \end{aligned} \tag{3}$$

For the generation of maximally entangled field state between two cavity modes such that if one mode has one photon then the other will be in vacuum, the atom after its interaction with the cavity fields, is required to be detected in ground state $|c\rangle$. This leads to the condition that probability amplitudes of the states $|c,1,0\rangle$ and $|c,0,1\rangle$ are equal, i.e., $\sin g_1t = \sin g_2t$, for $\theta = \pi/4$ and $\phi = 0$. The total probability of detecting the atom in ground state P_c is determined as $P_c = \left(\sin^2 g_1t + \sin^2 g_2t\right)/2$. This probability becomes maximum when the time of interaction of atom with mode A and mode B is $m\pi/2g_1$ and $n\pi/2g_2$, respectively. Here, m and n are odd integer numbers. Hence, in order to generate two mode entanglement the time of interaction of the atom with the cavity is odd integer multiple of half of the Rabi cycle. This ensures that the cavity will

obtain one photon in either of the two modes when atom is detected in ground state after its propagation through the cavity.

The interaction times of the atom with the two modes of the cavity field would be different because of the different coupling constants of each mode of radiation field. These interaction times of atom in the cavity can be controlled by using a velocity selector before the cavity and then applying Stark field adjustment so that atom becomes resonant with the cavity field modes only for the suggested period of time in each mode of the cavity field [3]. Hence the atom passing in the superposition of levels $|a\rangle$ and $|b\rangle$ interacts with the two cavity modes A and B for $m\pi/2g_1$ and $n\pi/2g_2$ interaction times, respectively. As a result the atom leaves the cavity in ground state and develops entangled states between the two cavity modes, viz.,

$$|\psi(A,B)\rangle = \frac{-i}{\sqrt{2}}\left[|0_A,1_B\rangle \pm e^{i\phi}\,|1_A,0_B\rangle\right], \tag{4}$$

by controlling the interaction time of the atoms with the cavities.

In order to generate the other two Bell bases we prepare the three level atom, in level $|a\rangle$ without considering the superposition of upper two levels, $|a\rangle$ and $|b\rangle$. We let the excited atom pass through two cavities successively, which are prepared initially in vacuum state. The transition from level $|a\rangle$ to $|c\rangle$ is again in resonance with cavity mode A, whereas the transition from $|b\rangle$ to $|c\rangle$ is in resonance with cavity mode B. We adjust the interaction time of the atom with first cavity field such that it sees a $\pi/2$ pulse. Hence, there occurs equal probability of finding the atom in ground state $|c\rangle$, after contributing one photon in the cavity mode A, and of finding the atom in the excited state $|a\rangle$, leaving the cavity mode in vacuum state. With a proper choice for the atomic dipole phase [6], we find an atom-field entanglement, such that,

$$|\psi_{at}(A,B)\rangle = \frac{1}{\sqrt{2}}\left[|a,0_A\rangle + |c,1_A\rangle\right] \otimes |0_B\rangle\,. \tag{5}$$

Before the atom enters the next cavity, we apply a laser field resonant to atomic transition $|b\rangle$ to $|c\rangle$. The width of the beam is adjusted such that the exiting atom from the first cavity field in the ground state $|c\rangle$, is pumped to excited state $|b\rangle$ with unit probability. However, if the exiting atom is in excited state $|a\rangle$ after interacting with the cavity mode A, the laser field will provide no excitation to the atom.

After passing through the laser field, the atom interacts with cavity mode B, which is initially in vacuum state. The interaction time of the atom with the field is adjusted such that the atom in the excited state $|b\rangle$ will be detected in ground state $|c\rangle$ with unit probability adding a photon in the cavity mode B. However, if the atom enters the cavity in the excited state $|a\rangle$, it will contribute no photon and will exit in the same atomic

state, leaving the cavity mode B in the vacuum state. Let the atom exiting the second cavity interact with another field, resonant with the atomic transition $|a\rangle \rightarrow |c\rangle$. The interaction time of $\pi/2$ and relative phase $\phi = \pi/2$ lead to $|a\rangle \rightarrow (\cos\theta|a\rangle - \sin\theta|c\rangle)$ and $|c\rangle \rightarrow (\cos\theta|a\rangle + \sin\theta|c\rangle)$ [7]. This makes the final state as

$$\begin{aligned} \left|\psi_{af}(A,B)\right\rangle &= (\cos\theta\,|0_A,0_B\rangle + \sin\theta\,|1_B,1_B\rangle)\otimes|a\rangle \\ &\quad + (\sin\theta\,|1_B,1_B\rangle - \cos\theta\,|0_A,0_B\rangle)\otimes|c\rangle. \end{aligned} \tag{6}$$

Detection of the atom in state $|a\rangle$ or $|c\rangle$ and choosing $\theta = \pi/4$, generates the entangled states of cavity field modes $\frac{1}{\sqrt{2}}(|0_A,0_B\rangle + |1_B,1_B\rangle)$ or $\frac{1}{\sqrt{2}}(|1_B,1_B\rangle - |0_A,0_B\rangle)$. Therefore, we find the entanglement of the two modes of radiation field with the atomic states, for a time larger than the life time of the propagating atom, as

$$|\psi(A,B)\rangle = \frac{1}{\sqrt{2}}\left[|1_B,1_B\rangle \pm |0_A,0_B\rangle\right], \tag{7}$$

which describes the entanglement of the two cavity field modes. The interaction times of the atoms with the cavity field mode A, laser field, and cavity field mode B are found as $m\pi/4g_1$, π/Ω, and $n\pi/2g_2$, respectively, where m and n are odd integers. Here Ω is the Rabi frequency of the laser field which interacts with the atom between the two cavities. If the relative difference of interaction times of atoms with the two cavities is π, we generate the entangled state with negative sign, given in equation (7). Hence, we can obtain the complete set of Bell basis by controlling the interaction times of the atom with the cavities in both schemes.

It should also be pointed out that our analysis assumes a perfectly isolated system. In a real experiment, the quantum motion of a trapped atom is obviously limited by sources of decoherence. In order to realize the suggested scheme in a laboratory experiment within the microwave region, we may consider slow Rb atoms in higher Rydberg states, which have lifetimes of the order of a few milliseconds. Finally, one may say that, the atomic decay rates, interaction times, and cavity lifetime ensure that the atom does not decay spontaneously. As this entanglement remains only for the cavity lifetime period, any application regarding this entangled state should be accomplished during this period [5].

We conclude our analysis by saying that in the present letter we have presented a powerful and useful tool to the generation of maximally entangled states using a three-level atom interacting with a bimodal cavity. We have demonstrated that, in many standard concepts and experiments in quantum optics where it appears necessary to use bimodal cavity, it is equally as valid to generate complete set of Bell basis states using sources of vacuum states.

In order to develop quantum universal logic gates able to generate maximum entanglement in the two cavity field modes, we consider a two-level circular Rydberg atom,

which passes through a high Q superconducting cavity. The cavity contains nondegenerate orthogonally polarized modes M_A and M_B with mode frequencies ω_A and ω_B. We consider the atomic transition frequency, ω_{eg} $(= \omega_e - \omega_g)$, in resonance with the field frequency ω_A. Here, ω_e and ω_g are the frequencies associated with excited state $|e\rangle$ and with the ground state $|g\rangle$, respectively. In presence of an electric field the excited state $|e\rangle$ changes to the state $|e'\rangle$ and the atomic transition frequency $\omega_{e'g}$ becomes equal to the electromagnetic field frequency ω_B due to the Stark effect [17]. Thus the atom emits a photon coherently in the cavity mode M_B at a different frequency, ω_B, in the presence of Stark field. Latter, the final state of the atom is analyzed in a state-selective field ionization detector. We show that by controlling the coherent interaction it is possible to realize two qubit quantum CNOT logic gate and a single Hadamard gate in the system.

The quantum Controlled NOT logic gate is a two-input two-output logic gate which requires a control qubit $|q_1\rangle$ and target qubit $|q_2\rangle$. The state of the control qubit $|\ q_1\rangle$ controls the state of the target qubit, $|\ q_2\rangle$, such that

$$|\ q_1\rangle\ |\ q_2\rangle\ \rightarrow |\ q_1\rangle\ |\ q_1 \oplus q_2\rangle,$$

where, $\oplus$ indicates addition modulo 2 [12]. This implies that the the target qubit is flipped if the control qubit carries logic one and remains unchanged if the control qubit carries logic zero.

As discussed above, in our study we take the control qubit as a two level atom, which is defined in a two dimensional Hilbert space with $|e\rangle$ and $|g\rangle$ as basis vectors. Here, $|e\rangle$ expresses the excited state of the two level atom, and $|g\rangle$ indicates its ground state.

The two non-degerate and orthogonally polarized cavity modes M_A and M_B make the target qubit, $|q_2\rangle$. The target qubit is defined in two dimensional Hilbert space spanned by the state vector $|\psi_1\rangle = |1_A, 0_B\rangle$, which expresses the presence of one photon in mode A, when no photon is present in mode B, and the state vector $|\psi_2\rangle = |0_A, 1_B\rangle$, which indicates that the mode A is in vacuum state, when one photon is present in the mode B.

Interaction of the two level atom, acting as a control qubit, with the electromagnetic cavity containing two field modes, acting as a target qubit, leads to the universal two-qubit control NOT logic gate and single bit Hadamard gate. We prepare the two level atom in the ground state, $|g\rangle$, in a Ramsey cavity. It enters an electromagnetic cavity which contains single photon of the field mode, M_A, whereas the mode M_B is in vacuum state. The transition frequency of the atom is taken equal to the frequency of the mode M_A, hence, the atom interacts resonantly with the field.

The interaction of the two level atom with the electromagnetic field mode, M_A is

described by the standard Jaynes-Cummings interaction Hamiltonian [17], expressed as

$$V = \hbar\mu_1(a^\dagger\sigma + \sigma^\dagger a),$$

where $a^\dagger$ (a) is field creation (annihilation) operator, $\sigma^\dagger = |e\rangle\langle g|$ $(\sigma = |g\rangle\langle e|)$ is atomic raising (lowering) operator, and μ_1 is the coupling constant. Hence the atom-field combined state of the system becomes,

$$|\psi\rangle = c_g|g, 1_A\rangle + c_e|e, 0_A\rangle. \tag{8}$$

The probability amplitudes, c_g and c_e, which govern the evolution of the atom initially in its ground state, change as a function of interaction time t such that,

$$c_g\left(t\right) = \cos\left(\Omega_A t/2\right), \tag{9}$$

and

$$c_e\left(t\right) = -i\sin\left(\Omega_A t/2\right). \tag{10}$$

The frequency, $\Omega_A = 2\mu_1\sqrt{n_A}$, describes the Rabi frequency of the atom in the mode M_A containing n_A number of photons. The atom interacts for a time π/Ω_A with mode M_A and completes half of the Rabi oscillation. As a result it absorbs the cavity photon in mode M_A and jumps to its excited state, $|e\rangle$.

After the interaction time π/Ω_A, we apply a Stark field which shifts the excited state from $|e\rangle$ to $|\acute{e}\rangle$. Hence the atom observes a change in its transition energy and finds itself resonant with the mode, M_B. The atom interacts with the cavity mode, M_B, resonantly and follows the interaction Hamiltonian, viz.,

$$V = \hbar\mu_2(b^\dagger\hat{\sigma} + \hat{\sigma}^\dagger b)$$

where $b^\dagger$ (b) is creation (annihilation) operator of the field mode M_B, $\hat{\sigma}^\dagger = |e\rangle\langle g|$ $(\sigma = |g\rangle\langle e|)$ atomic raising (lowering) operator, and μ_2 is the coupling constant. The probability amplitudes of the resonant atom flips as,

$$c_e\left(t\right) = \cos\left(\Omega_B(t - t_0)/2\right), \tag{11}$$

and

$$c_g\left(t\right) = -i\sin\left(\Omega_B(t - t_0)/2\right), \tag{12}$$

where $t > t_0 = \pi/\Omega_A$ and $\Omega_B = 2\mu_2\sqrt{n_B + 1}$, where n_B describes the number of photons in cavity B. After an interaction with the mode M_B for a time $\pi(\Omega_A + \Omega_B)/\Omega_A\Omega_B$, the atom leaves the cavity in its ground state $|g\rangle$ and thus contributes one photon to the cavity

mode M_B. Therefore, the atom performs a swapping of electromagnetic fields between two field modes by a controlled interaction.

In case we prepare the two level atom in its ground state, $\mid g\rangle$, when the electromagnetic cavity contains the mode M_A in its vacuum state and a single photon in electromagnetic field mode, M_B. The atom becomes resonant with the electromagnetic cavity field mode, M_B, in the presence of the Stark field at time, $t = 0$. It exhibits a controlled interaction with the field mode M_B, for a time π/Ω_B which is equal to half of the Rabi oscillation time. Here, $\Omega_B = 2\mu_2\sqrt{n_B}$. Thus the state of the system at $t = \pi/\Omega_B$ becomes $|e, 0, 0\rangle$. We switch off the Stark field and let the atom interact resonantly with mode M_A for a time $\pi(\Omega_A + \Omega_B)/\Omega_A\Omega_B$, where $\Omega_A = 2\mu_1\sqrt{n_A + 1}$. Therefore, the atom again leaves the cavity in its ground state $|g\rangle$, and performs field swapping by contributing one photon to the field mode, M_A.

The target qubit made up of the electromagnetic fields remains unchanged if the control qubit, that is the two level atom is initially in its excited state. The atom prepared in its excited state $|e\rangle$ interacts with the mode M_A containing one photon field. The Rabi oscillation frequency is given by $\Omega_A = 2\mu_1\sqrt{n_A + 1}$ thus it completes one Rabi oscillation in time $2\pi/\Omega_A = \sqrt{2}\pi/\mu_1$. The atom leaves the cavity in state, $|e\rangle$, without disturbing the field mode, M_A. However, if the cavity field mode M_B is in Fock state one, the atom becomes resonant with M_B in the presence of a Stark field to introduce zero detuning between atomic transition frequency and the field frequency. Hence, the atom completes one Rabi oscillation in the field for the interaction time $\sqrt{2}\pi/\mu_2$ and leaves the cavity in its excited state $|e\rangle$ without contributing to the radiation field mode, M_B.

Hadamadard gate generates superposition state of a qubit provided the system is in one of the bases vectors of the two dimensional Hilbert space. Hence, we consider the control qubit in state $|g\rangle$ and target qubit in the electromagnetic field state $|\psi_1\rangle = |1_A, 0_B\rangle$. The atom interacts with the cavity for a time equal to one fourth of Rabi oscillation period, that is $\pi/2\Omega_A$. Thus the combined atom-field state of the system becomes, $(|e, 0_A, 0_B\rangle + |g, 1_A, 0_B\rangle)/\sqrt{2}$. Later, we introduce a Stark field, which shifts the excited state from $|e\rangle$ to $|e\rangle$. Thus the atom interacts in resonance with the mode M_B. The atom leaves the cavity after time π/Ω_B, in its ground state, $|g\rangle$, and contributes one photon to the field in mode, M_B. The resultant state of the system becomes a superposition state of $|\psi_1\rangle$ and $|\psi_2\rangle$, with an equal probability to find the system in either of the two bases vectors.

In this paper we present a scheme to engineer two-qubit quantum controlled NOT logic gate and single bit Hadamard gate by a controlled interaction between two-mode, high Q, electromagnetic cavity field and a two-level atom. For the purpose we take the

two-level atom as control qubit, whereas the target qubit is made up of two modes of the cavity field. We express the development of quantum controlled NOT logic gate in Table 1, as,

$\|q_1\rangle$	$\|q_2\rangle$	$\|q_1\rangle$	$\|q_1 \oplus q_2\rangle$
$\|g\rangle$	$\|1_A, 0_B\rangle$	$\|g\rangle$	$\|0_A, 1_B\rangle$
$\|g\rangle$	$\|0_A, 1_B\rangle$	$\|g\rangle$	$\|1_A, 0_B\rangle$
$\|e\rangle$	$\|1_A, 0_B\rangle$	$\|e\rangle$	$\|1_A, 0_B\rangle$
$\|e\rangle$	$\|0_A, 1_B\rangle$	$\|e\rangle$	$\|0_A, 1_B\rangle$

In order to realize our scheme in laboratory, we may fellow the experimental setup of the Ref. [16]. We consider circular Rydberg rubidium atom, with principal quantum number 51 and 50 acting as levels $|e\rangle$ and $|g\rangle$, respectively, as control bit. The transition from level $|e\rangle$ to $|g\rangle$ occurs at frequency 51.1 GHz. A very small rate of injection makes the probability of having two atoms at the same time in the cavity very small. The optical cavity is a Fabry-Perot resonator made of two spherical nibium mirrors. The two orthogonally polarized TEM_{900} modes, M_A and M_B have the same Gaussaian geometry with waist 6 mm. The frequency splitting occurs due to a slight mirror shape anisotropy. The photon damping times are $T_{r,a} = 1\ ms$ and $T_{r,b} = 0.9\ ms$ for the electromagnetic field modes M_A and M_B, respectively.

The engineering of the universal logic gate helps us to generate multi-qubit quantum gates [18] and to engineering entanglement, in various modes of cavity field [16, 3]. Furthermore, the experimental accessability of these logic gates make it possible to implement quantum algorithms in the QED setup. We may use the system for secret data communication i.e, cryptography, and to perform teleportation.

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